

The 7th International Symposium on
Generalized Convexity/Monotonicity

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Hanoi Institute of Mathematics

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Topics

The symposium is aimed at bringing together researchers from all continents to report their latest results and to exchange new ideas in the field of generalized convexity and generalized monotonicity and their applications in optimization, control, stochastic, economics, management science, finance, engineering and related topics.

Organizers

- International Working Group on Generalized Convexity (WGGC)
- Institute of Mathematics, Vietnam NCST

Location

The conference takes place at the Institute of Mathematics
National Centre for Natural Sciences and Technology (NCST)
18 Hoang Quoc Viet Street, Cau Giay District, Hanoi, Vietnam.

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Abstracts

Stability of generalized convex functions

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Some kind of generalized convexity is said to be stable (or absolutely stable, respectively) with respect to some property (P) if this property is maintained during an arbitrary function from this class is disturbed by a continuous linear functional with sufficiently small norm (or by an arbitrary linear functional, respectively). In this paper we consider some main properties of generalized convex functions:

(P_l) Each lower level set is convex,

(P_m) Each local minimizer is a global minimizer,

(P_m^γ) Each γ -local minimizer x^* of a function $f(\cdot) : D \rightarrow \mathbb{R}^1$ defined by $f(x^*) \leq f(x)$ for all $x \in D$ with $\|x - x^*\| \leq \gamma$ is a global minimizer.

There arise the questions: What kinds of generalized convexities are stable or absolutely stable with respect to (P_l), (P_m) or (P_m^γ)?

We prove that known generalized convexities like quasiconvexity, explicit quasiconvexity and pseudoconvexity are not stable with respect to (P_l) or (P_m). The notion of s -quasiconvex functions is introduced which is stable with respect to (P_l) and (P_m). Moreover, only classical convexity is absolutely stable with respect to (P_l) and (P_m). Additionally, we prove that so-called γ -convexlike functions are absolutely stable with respect to (P_m^γ) and if a lower semi-continuous function $f(\cdot) : [a, b] \subset \mathbb{R}^1 \rightarrow \mathbb{R}^1$ is absolutely stable with respect to (P_m^γ) then it must be γ -convexlike.

The Banach Iterative Procedure for Solving Monotone Variational Inequalities

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A function (mapping) $F : C \rightarrow C$ is said to be strongly Lipschitzian on C with constant $\delta > 0$ if

$$\|F(x) - F(x')\|^2 \leq \delta \langle x - x', F(x) - F(x') \rangle \quad \forall x, x' \in C.$$

We consider the variational inequality

$$\text{find } x^* \in C : \langle F(x^*) - x - x^* \rangle \geq 0 \quad \forall x \in C. \quad (1)$$

where $C \subset R^n$ is a closed convex set and F is a Lipschitzian mapping from C to R^n .

We use the differentiable merit function recently developed by Fukushima for solving variational inequality (1). We show that one can choose a suitable regularization matrix such that a solution of the variational inequality can be found by computing a fixed point of a certain nonexpansive mapping. This result allows that the Banach iterative procedure for contractive mapping can be used for solving variational inequality with strongly Lipschitzian function. Our method gives a new look for the Auxiliary Principle for monotone variational inequalities.

The system of (generalized) vector equilibrium problems and its applications

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We introduce the system of vector equilibrium problems and prove the existence of a solution. As an application we derive some existence results for a system of vector variational inequalities. We also establish some existence results for a system of vector optimization problems which includes the Nash equilibrium problem with vector-valued functions.

In a follow-up study we introduce the system of generalized vector equilibrium problems with its various realizations for variational inequalities and optimization models. By using a maximal element theorem we establish existence results for such a system. As an application we derive existence results for a solution of a more general Nash equilibrium problem with vector-valued functions. Results are obtained under suitable generalized convexity assumptions.

Normal operator in quasiconvex programming

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Let us consider the optimization problem

$$(P) \text{ Minimize } f(x) \text{ with } x \in K$$

where X is a Banach space
 K is a closed convex subset of X
 f is a nondifferentiable quasiconvex function.

Using some recent existence results for variational inequality problems and the particular properties of the normal operator we obtain an existence theorem and a uniqueness theorem for problem (P). The Banach space X is not supposed to be reflexive, nor K to be compact.

Some other interesting properties of the normal operator will be presented.

On the equivalence of several generalizations of the Ekeland variational principle and the original principle

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The Ekeland variational principle is one of the most important principles in nonlinear analysis which appeared in the last three decades. There have been numerous applications in optimization, fixed point theory, critical point theory.... Also, many generalizations of the principle have been proposed for more general settings (e.g. for vector optimization, and for multivalued optimization) and for additional or deeper conclusions. Recently, in [1], [2] the authors proved several generalized principles, containing additional properties of the approximate extremal point which is asserted to exist in the Ekeland principle. In the present note we prove that these generalizations are in fact equivalent to the original principle.

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A finite iteration technique for a fuzzy quadratic programming problem

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In this paper we consider two problems, one under symmetric fuzzy environment, and the second under non-symmetric fuzzy environment, such that each problem has a single fuzzy quadratic objective function and a number of fuzzy and crisp linear constraints. To solve such a problem, we suggest a finite step method that uses linear programming and parametric quadratic programming approach. Furthermore, we present a numerical example to demonstrate the method developed.

Inf-Invex Alternative Theorem With Application To Vector Valued Games

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The definition of inf-invexity is extended for a function of two variables and is then utilized to prove an inf-invexity alternative theorem of Gordan type. As

an application of this, a vector valued non-linear constrained game is shown to be equivalent to a pair of symmetric dual multiobjective non-linear programming problems in which the multiplier corresponding to the objective is a vector valued function of two variables.

On pseudomonotone maps T for which $-T$ is also pseudomonotone

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Recently pseudomonotone variational inequalities have been studied quite extensively, hereby extending the theory of pseudoconvex minimization problems. The focus of the present work are "pseudoaffine maps", i.e., pseudomonotone maps T for which $-T$ is also pseudomonotone. A particular case of such maps are the gradients of pseudolinear functions, which also have been studied extensively. Our main goal is to derive the general form of pseudoaffine maps which are defined on the whole space.

Testing a global optimization method for D.C. programming problems

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A method for modeling a real constrained optimization problem as a D.C. canonical programming problem has been developed from a new procedure of D.C. representation of any polynomial function. A modified algorithm of D.C. programming has been implemented to solve this problem. The solution obtained with a local optimization package is also included and their results are compared.

A Logarithmic Barrier Strategy for Vector Optimization

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We propose a logarithmic penalization method for finding efficient points of nonsmooth constrained convex vector-valued problems. Under the sole assumption of the existence of a Slater-type point, we show that the technique produces a uni-parametric point-to-set mapping, whose nonempty outer limit at zero is contained in the dual optimal set, and such that the primal projection of this limit is included in the primal optimal set.

Combinatorial Optimization and Convexity

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(Abstract to be given later).

Pseudoconvexity under the Charnes-Cooper transformation

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Charnes and Cooper (1962) reduced a linear fractional program to a linear program with help of a suitable transformation of variables. We show that this transformation preserves pseudoconvexity of a function. The result is then used to characterize sums of two linear fractional functions which are still pseudoconvex. This in turn leads to a characterization of pseudolinear sums of two linear fractional functions.

Duality for fractional optimization with set constraints

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(Abstract to be given later)

Duality in multiobjective optimization problems with set constraints

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We propose four different duality problems for a vector optimization program with both set constraints and inequality constraints. For all dual problems we state weak and strong duality theorems based on various generalized concavity assumptions. The proposed dual problems provide a unified framework generalizing Wolfe and Mond-Weir results.

On convex relaxations for nonconvex quadratic programs

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The aim of this paper is to suggest a branch-and-bound scheme, based on convex relaxations of the objective function, to solve nonconvex quadratic programs over a compact feasible region.

In order to obtain the various relaxations, the quadratic objective function is decomposed in different d.c. forms.

To improve the tightness of the relaxations, we finally suggest to use a particular decomposition and to solve the corresponding relaxed problems with an algorithm based on the so called "optimal level solutions" parametric approach.

Convexifactors, approximate Jacobians and optimality conditions

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(Abstract to be given later)

Characterizing invex and related properties

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A characterization of invex, given by Glover and Craven, is extended to functions in abstract spaces. Pseudoinvex for a vector function coincides with invex in a restricted set of directions. The V-invex property of Jeyakumar and Mond is also characterized. Some differentiability properties of the invex scale function are also obtained.

Convex Minty vector variational inequality

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In the past decade, the results concerning variational inequalities, both of Minty and Stampacchia type, have been adapted to the vector inequalities introduced by Giannessi in [3] (Stampacchia type) and in [4] (Minty type).

Since the early beginning, they have been applied to the study of vector optimization problems. However some lacks arise when comparing the results obtained for scalar inequalities and the new ones presented in [4]. Mainly it is possible to see that some more convexity assumptions are due to prove a (integrable) Minty vector inequality is a sufficient condition for efficiency, while the same results hold trivially without assumptions for scalar Minty inequalities.

We will first show that such a gap could not be filled, and we deduce that a stronger solution concept is what we need to solve the problem. To do that we apply some convexification starting from the definition of solution given in [4] to get a new Minty type vector variational inequality which shows to be, in the integrable case, a sufficient condition for efficient minimizers without convexity of the objective function. Moreover we prove the result in [4] follows from our as a special case.

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Mollified derivatives and second-order optimality conditions

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In this talk we present an extension to the second-order of the approach introduced by Craven [3] and by Ermoliev, Norkin and Wets [4] to define generalized derivatives even for discontinuous functions, which often arise in applications (see [4] for references). To deal with such problems a number of approaches have been proposed to develop a subdifferential calculus for nonsmooth and even discontinuous functions. Among the many possibilities, let us remember the notions due to Aubin [1], Clarke [2], Ioffe [6], Michel and Penot [10], Rockafellar [12], in the context of Variational Analysis. The previous approaches are based on the introduction of first-order generalized derivatives. Extensions to higher-order derivatives have been provided for instance by Hiriart-Hurruty, Strodiot and Hien Nguyen [5], Jeyakumar and Luc [7], Klatter and Tammer [8], Michel and Penot [9], Yang and Jeyakumar [15], Yang [16]. Most of these higher-order approaches assume that the functions involved are of class $C^{1,1}$, that is once differentiable with locally Lipschitz gradient, or at least of class C^1 . Anyway, another possibility, concerning the differentiation of nonsmooth functions dates back to the 30s and is related to the names of Sobolev [14], who introduced the concept of “weak derivative” and later of Schwartz [13] who generalized Sobolev’s approach with the “theory of distributions”. These techniques are widely used in the theory of partial differential equations, in Mathematical Physics and in related problems, but they have not been applied to deal with optimization problems involving nonsmooth functions, until the work of Ermoliev, Norkin and Wets. The tools which allow to link the “modern” and the “ancient” approaches to Nonsmooth Analysis are those of “mollifier” and of “mollified functions”. More specifically, the approach followed by Ermoliev, Norkin and Wets appeals to some of

the results of the theory of distributions. They associate with a point $x \in \mathbb{R}^n$ a family of mollifiers (density functions) whose support tends toward x and converges to the Dirac function. Given such a family, say $\{\psi_\varepsilon, \varepsilon > 0\}$, one can define a family of mollified functions associated to a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ as the convolution of f and ψ_ε (mollified functions will be denoted by f_ε). Hence a mollified function can be viewed as an averaged function. The mollified functions possess the same regularity of the mollifiers ψ_ε and hence, if they are at least of class C^2 , one can define first and second-order generalized derivatives as the cluster points of all possible values of first and second-order derivatives of f_ε . For more details one can see [4].

We remember also that an approach based on similar techniques has been used to solve nonsmooth equations, with the introduction of smoothing functions and smoothing Newton methods [11].

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Closedness and Connectedness of Efficient Solution Set in \mathcal{S} -Strictly Quasiconcave Vector Maximization

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In this paper, we discuss the relationship among the concepts of an \mathcal{S} -strictly quasiconcave vector-valued function introduced by Benson and Sun, a \mathcal{C} -strongly quasiconcave vector-valued function and a \mathcal{C} -strictly quasiconcave vector-valued function in a topological vector space with a lattice ordering. We generalize a main result obtained by Benson and Sun about the closedness of an efficient solution set in multiple objective programming. We prove that an efficient solution set is closed and connected when the objective function is a continuous \mathcal{S} -strictly quasiconcave vector-valued function, the objective space is a topological vector lattice and the ordering cone has a nonempty interior.

Convexity and the core for set games and cooperative games: an equivalence theorem

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Let the universe \mathcal{U} denote an abstract set which is fixed throughout the remainder. Following various introductory papers, a set game is a pair $\langle N, v \rangle$, where N is a nonempty, finite set, called player set, and $v : 2^N \rightarrow 2^{\mathcal{U}}$ is a characteristic mapping, defined on the power set of N , satisfying $v(\emptyset) := \emptyset$.

An element $i \in N$ and a nonempty subset $S \subseteq N$ (or $S \in 2^N$ with $S \neq \emptyset$) is called a player and coalition respectively, and the associated set $v(S) \subseteq \mathcal{U}$ is called the worth of coalition S , to be interpreted as the (sub)set of items from \mathcal{U} that can be obtained (are needed, preferred, owned) by coalition S if its members cooperate.

Concerning the solution theory for set games, a solution ψ on the class of set games associates a so-called allocation $\psi(N, v) = (\psi_i(N, v))_{i \in N} \in (2^{\mathcal{U}})^N$ with every set game $\langle N, v \rangle$. The so-called set allocation $\psi_i(N, v) \subseteq \mathcal{U}$ to player i in the set game $\langle N, v \rangle$ represents the items that are given, according to the solution ψ , to player i from participating in the game.

Concerning the yet undeveloped solution theory for set games, the paper focuses on the core concept for a special family of set games called convex set games. The main equivalence theorem states that, for the convexity of a monotonic set game,

it is necessary and sufficient that a large number of appropriately chosen set allocations belong to the core of the set game. These so-called marginalistic worth set allocations are constructed with the help of orderings of the fixed player set, the orderings of which may be classified in two types, called even or odd orderings. The main equivalence theorem resembles a similar one in the context of convex cooperative games with transferable utility. Recall that a cooperative TU-game with player set N is described by a characteristic mapping $w : 2^N \rightarrow \mathbb{R}$, where the (positive) real number $w(S)$ usually is interpreted as the monetary worth of the benefits achieved by coalition S if its members cooperate (e.g., in a joint venture). The convexity notion for cooperative TU-games is very well established since its introduction in 1971 by Shapley.

Some Applications of Generalized Convexity/Nonsmooth Analysis techniques to Second Order PDEs

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A central theme of the theory of second order elliptic partial differential is the so-called comparison principle. A general second order elliptic partial differential equation is of the form

$$\begin{aligned} F(x, u(x), Du(x), D^2u(x)) &= 0 \quad \text{for } x \in \Omega \\ u(x) &= g(x) \quad \text{for } x \in \delta\Omega \end{aligned} \tag{1}$$

where $Du(x)$, $D^2u(x)$ are the first and second order derivatives of u (possibly in a generalized sense) and g denotes the boundary condition which the solution u must satisfy on the boundary $\delta\Omega$ of the domain Ω . The function $F(x, r, p, Q) : \mathbb{R}^n \times \mathbb{R} \times \mathbb{R}^n \times \mathcal{S}(n) \rightarrow \mathbb{R}$ usually has to satisfy some structure condition. Unfortunately such a nonlinear equation may not admit classical solution (i.e. a function $u \in C^2(\mathbb{R}^n)$ that satisfies the equation pointwise). Thus an array of generalized solution have been proposed by a number of authors. One such concept is the so-called viscosity solution which (when it exists) may be

constructed as the supremum of the class of subsolutions of (1) which also satisfy the boundary condition. We say u is a sub (super) solution of (1) when

$$F(x, u(x), p, Q) \leq (\geq) 0 \quad \text{for } x \in \Omega$$

for all $(p, Q) \in \partial^{2,+}u(x) = -\partial^{2,-}(-u)(x)$ (or $(p, Q) \in \partial^{2,-}u(x)$ for super solution). Here $\partial^{2,-}w$ denotes the so called subjet (or second order viscosity subdifferential) defined by

$$\partial^{2,-}w(x) := \{(p, Q) = (\nabla\varphi(x), \nabla^2\varphi(x)) \mid f - \varphi \text{ has a local minimum at } x\}.$$

We say comparison holds if whenever u is a super solution and v is a subsolution with $u(x) \geq v(x)$ for $x \in \delta\Omega$ then $u \geq v$ on all of Ω . To establish this results one need to investigate the maximizing point of $v - u$ (and show it is non-positive). In this talk we will discuss two different ways of establishing comparison. One approach uses composite supremal and infimal convolutions, like the Lasry-Lions double convolution, to provide a $C^{1,1}(\mathbb{R}^n)$ approximation of the possibly nonsmooth functions u and v . Unlike earlier work we provide explicit formula describing how the subdifferential information of the function is effected by such a smoothing processes, correcting an error in the literature. The second method takes its motivation form the study of the subdifferential of the difference of two functions to provide a new proof a fuzzy sum formula for the difference of two nonsmooth functions. This has some novel character and allows comparison to be established under weaker assumptions than are usually used. Some discussion of applications will be given and future directions for research.

Criteria for the convexity behavior of homogeneous functions

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Homogeneous functions with certain differentiability properties satisfy a system of partial differential equations. For such functions being twice continuously differentiable on open convex sets we give necessary and/or sufficient conditions

for different kinds of convexity. Homogeneous functions are extensively used not only in the economics literature but also in the theory of inequalities and optimization. Relationships to these subjects will be presented.

Epsilon-optimality for nonsmooth programming on a Banach space

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We establish Lagrange multiplier rule, in terms of generalized gradient, that characterizes the epsilon-efficiency of a nonsmooth programming problem(s) on a real Banach space. This rule is then utilized to establish relationship between epsilon-efficient solution of program(s) and generalized saddle point under appropriate conditions.

Approximate Optimization of Convex Set Functions

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We introduce the notion of epsilon-subdifferential for convex set function and discuss some of its properties. These properties are then utilized to derive epsilon-Pareto optimality conditions of Karush-Kuhn-Tucker type for non-differentiable multiobjective optimization problem with convex set functions.

epsilon-optimality without constraint
qualification for multiobjective
fractional program

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In this paper, we concentrate on a non-differentiable multiobjective fractional programming problem subject to convex inequality constraints, equality constraints, and abstract constraints. Epsilon-parametric technique is used to transform the given problem into non-differentiable multiobjective programming problem. Subsequently, we employ the exact penalty function approach to derive the Karush-Kuhn-Tucker type necessary and sufficient optimality conditions for the given problem without using any constraint qualification.

Invexity of supremum and infimum functions and
applications

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Let Q be a metricable compact topological space, and for all $\alpha \in Q$, let f_α be a real-valued function defined on \mathbb{R}^n . Consider the following functions

$$f(x) = \sup_{\alpha \in Q} f_\alpha(x), \quad g(x) = \inf_{\alpha \in Q} f_\alpha(x).$$

Under suitable assumption we derive the results ensuring the functions f and g are invex when all the functions f_α ($\alpha \in Q$) are invex. Applying these results to a class of mathematical programs, optimality conditions are established under suitable invexity hypotheses.

On maximality, continuity and single-valuedness of pseudomonotone maps

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We define the concept of "D-maximal pseudomonotone map", where "D" stands for "domain". Conditions for a pseudomonotone map to be D-maximal are given; in particular, it is shown that the Clarke subdifferential of a locally Lipschitz pseudoconvex function is D-maximal pseudomonotone. Finally, the continuity and single-valuedness of pseudomonotone maps are investigated.

Boundedness and continuity of γ -convex functions in normed spaces

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For a fixed positive number γ , a real-valued function f defined on a convex subset D of a normed space X is said to be γ -convex if it satisfies the inequality

$$f(x'_0) + f(x'_1) \leq f(x_0) + f(x_1) \quad \text{for } x'_i \in [x_0, x_1], \quad \|x'_i - x_i\| = \gamma, \quad i = 0, 1,$$

whenever $x_0, x_1 \in D$ and $\|x_0 - x_1\| \geq \gamma$. This condition implies that the Jensen inequality

$$f(x_\lambda) \leq (1 - \lambda)f(x_0) + \lambda f(x_1), \quad x_\lambda := (1 - \lambda)x_0 + \lambda x_1 \quad (1)$$

holds at $x_\lambda = x'_0$ or $x_\lambda = x'_1$. This condition is rather weak, nevertheless γ -convex functions possess some interesting analytical properties, which are presented in our talk. For instance,

- if there is some $x_* \in D$ such that f is bounded below on $D \cap \bar{B}(x_*, \gamma)$, then so is it on each bounded subset of D ;

- if f is bounded on some closed ball $\bar{B}(x_*, \gamma/2) \subset D$ and D' is a bounded subset of D , then f is bounded on D' iff it is bounded above on the boundary of D' ;

- if $\dim X > 1$ and the interior of D contains a closed ball of radius γ then f is either locally bounded or nowhere locally bounded on the interior of D ;

- if D contains some open ball $B(x_*, \gamma/2)$ in which f has at most countably many discontinuities, then f possesses at most countably many discontinuities on each line through x_* , which implies that the set of all points at which f is continuous is dense in D .

As a particular kind, f is called symmetrically γ -convex if it satisfies the Jensen inequality (1) at both $x_\lambda = x'_0$ and $x_\lambda = x'_1$. Such a function has stronger analytical properties. For example, if X is a finite-dimensional normed space then a symmetrically γ -convex function f on $D \subset X$ is locally Lipschitzian at any so-called γ -interior point x of D defined by $B(x, r) \subset D$ for some $r > \gamma$.

Some quasi-physical algorithm on optimization research

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(Abstract to be given later)

Generalized variational inclusions with generalized m-accretive mappings

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(Abstract to be given later)

Contractibility of the solution set of a semistrictly quasiconcave vector maximization problem

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(Abstract to be given later)

Non-Archimedean Solutions for Linear Inequality Systems

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Let S be an arbitrary set of vectors in \mathbf{R}^{n+1} . Our problem is to find a necessary and sufficient condition for the existence of a solution (x_1, x_2, \dots, x_n) to the following system of linear inequalities :

$$a_1x_1 + a_2x_2 + \dots + a_nx_n < b \quad \text{for all } (a_1, a_2, \dots, a_n, b) \in S. \quad (A)$$

If S is a finite set, the separating hyperplane lemma gives the well-known condition: there exists a solution to (A) if and only if $\mathbf{0}$ is not a linear convex combination of the vectors in $S \cup \{(0, \dots, 0, 1)\}$. In this talk we show that the same condition can also be applied to the infinite case if we adopt a suitable non-Archimedean structure as the domain of solutions to linear inequality systems.

For this purpose we introduce an extended structure of the reals \mathbf{R} with an infinitesimal ϵ , that is, a solution of the system " $-x < 0$ and $x < 1/k$ for all $k \in \mathbf{N}$." We show that this extended structure coincides with the set of lexicographically ordered vectors. The main result is derived as a consequence of the lexicographical separation theorem (see [1], [2]) that any two disjoint convex sets in \mathbf{R}^n can be separated lexicographically.

Some applications in utility theory are also presented.

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Inexact variants of the proximal point method without monotonicity

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We study some variants of the proximal point method for finding zeroes of operators. We are interested in the case of procedures with inexact iterates, which allow for constant relative errors, in the line of the recently proposed hybrid proximal-extragradient algorithm. We consider the case of nonmonotone operators. Generalizing the recent work of Pennanen, who dealt only with the case of exact solution of the proximal subproblems, we establish local convergence when the operator is ρ -hypomonotone, provided the regularization coefficients are greater than 2ρ . We also prove linear convergence rate when, additionally,

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the inverse of the operator is locally Lipschitz continuous near 0. Finally, as an application of these results, we present new inexact multiplier methods for a rather general family of problems, including variational inequalities and constrained optimization problems.

Convergence of iteration processes for nonexpansive mappings in Banach spaces

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Let E be a reflexive Banach space with a uniformly Gâteaux differentiable norm, and S a mapping of the form

$$S = \alpha_0 I + \alpha_1 T_1 + \alpha_2 T_2 + \cdots + \alpha_k T_k,$$

where $\alpha_i \geq 0$, $\alpha_0 > 0$, $\sum_{i=0}^k \alpha_i = 1$ and $T_i : E \rightarrow E$ ($i = 1, 2, \dots, k$) is a nonexpansive mapping. For an arbitrary $x_0 \in E$, let $\{x_n\}$ be a sequence in E defined by an iteration $x_{n+1} = Sx_n$, $n = 0, 1, 2, \dots$. We establish a dual weak almost convergence result of $\{x_n\}$ in a reflexive Banach space with a uniformly Gâteaux differentiable norm. As a consequences of the result, a weak convergence result of $\{x_n\}$ is also given.

Generalized semi-pseudomonotone set-valued variational-type inequality

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In [1], Chen considered the following variational inequality (P-1) for a semi-monotone single-valued map $A : K \times K \rightarrow E^*$, where K is a bounded closed convex subset of E^{**} , the second dual of a real Banach space E .

(P-1) Find an $x \in K$ such that

$$\langle A(x, x), y - x \rangle \geq 0 \quad \text{for all } y \in K.$$

And then, very recently Fang and Huang [2] considered the following generalized variational-type inequality (P-2) for a semi-monotone single-valued map $A : K \times K \rightarrow E^*$.

(P-2) Find an $x \in K$ such that

$$\langle A(x, x), \eta(y, x) \rangle + f(y) - f(x) \geq 0 \quad \text{for all } y \in K,$$

where $\eta : K \times K \rightarrow E^{**}$ is a map and $f : K \rightarrow \mathbb{R} \cup \{\infty\}$ is a function.

In 2000, Kassay and Kolumban [4] considered the existence of solutions to the following variational inequalities (P-3) and (P-4) for semi-pseudomonotone set-valued maps $A : K \times K \rightarrow 2^E$, where K is a nonempty convex subset of E^* .

(P-3) Find an $x \in K$ such that

$$\sup_{u \in A(x, x)} \langle u, y - x \rangle \geq 0 \quad \text{for all } y \in K.$$

(P-4) (Minty-type problem)

Find an element $x \in K$ such that

$$\sup_{u \in A(y, x)} \langle u, y - x \rangle \geq 0 \quad \text{for all } y \in K.$$

In this paper, we consider the existence of solutions to the following variational-type inequality for a relaxed α -semi-pseudomonotone set-valued map $A : K \times K \rightarrow 2^{E^*}$, where K is a nonempty closed convex subset of E^{**} ;

Find $x \in K$ such that for each $y \in K$ there exists $u \in A(x, x)$ satisfying

$$\langle u, \eta(y, x) \rangle + f(y, x) \geq 0. \tag{1}$$

Definition 1 Let K be a nonempty subset of E^{**} . A set-valued map $A : K \times K \rightarrow 2^{E^*}$ is said to be relaxed α -semi-pseudomonotone if the following conditions hold;

- (a) for each fixed $w \in K$, $A(w, \cdot) : K \rightarrow 2^{E^*}$ is relaxed α -pseudomonotone, i.e., there exists a function $\alpha : E^{**} \rightarrow \mathbb{R}$ with $\alpha(tz) = k(t)\alpha(z)$ for $z \in E^{**}$, where $k : (0, 1) \rightarrow (0, 1)$ is a function with $\lim_{t \rightarrow 0} \frac{k(t)}{t} = 0$, such that for every pair of points $x, y \in K$ and for all $u \in A(w, x)$, $v \in A(w, y)$, we have

$$\langle u, \eta(y, x) \rangle + f(y, x) \geq 0$$

implies

$$\langle v, \eta(y, x) \rangle + f(y, x) \geq \alpha(y - x),$$

where $\eta : K \times K \rightarrow E^{**}$ is a map and $f : K \times K \rightarrow \mathbb{R}$ is a function.

- (b) for each fixed $y \in K$, $A(\cdot, y)$ is completely continuous, i.e., for any net $\{x_\beta\}$, if $x_\beta \rightarrow x_0$ and $u_0 \in A(x_0, y)$, then there exists a net $\{u_\beta\}$ in E^* such that $u_\beta \in A(x_\beta, y)$ for each β , which has a convergent subnet to u_0 in the norm topology of E^* , where \rightarrow denotes the weak* convergence in E^{**} .

Theorem 1 Let E be a real Banach space and K a nonempty bounded closed convex subset of E^{**} . Let $A : K \times K \rightarrow 2^{E^*}$ be a relaxed α -semi-pseudomonotone set-valued map, and $f : K \times K \rightarrow \mathbb{R} \cup \{+\infty\}$ a proper function such that

- (i) for fixed $v \in E^*$, $x \mapsto \langle v, \eta(x, \cdot) \rangle + f(x, \cdot)$ is linear, weakly lower semicontinuous,
- (ii) $\eta(x, y) + \eta(y, x) = \bar{0}$ and $f(x, y) + f(y, x) = 0$ for $x, y \in K$,
- (iii) $\alpha : E^{**} \rightarrow \mathbb{R}$ is convex, weakly lower semicontinuous, and
- (iv) for each $x \in K$, $A(x, \cdot) : K \rightarrow 2^{E^*}$ is finite dimensional continuous.

Then problem (1) is solvable.

Theorem 2 Let E be a real Banach space and K a nonempty unbounded closed convex subset of E^{**} . Let $A : K \times K \rightarrow 2^{E^*}$ be a relaxed α -semi-pseudomonotone set-valued map, and $f : K \times K \rightarrow \mathbb{R} \cup \{+\infty\}$ a proper function such that

- (i) for fixed $v \in E^*$, $x \mapsto \langle v, \eta(x, \cdot) \rangle + f(x, \cdot)$ is linear, lower semicontinuous,
- (ii) $\eta(x, y) + \eta(y, x) = \bar{0}$ and $f(x, y) + f(y, x) = 0$ for $x, y \in K$,
- (iii) $\alpha : E^{**} \rightarrow \mathbb{R}$ is convex, weakly lower semicontinuous,
- (iv) for each $x \in K$, $A(x, \cdot) : K \rightarrow 2^{E^*}$ is finite dimensional continuous, and
- (v) there exists an $x_0 \in K$ such that

$$\underline{\lim}_{\|x\| \rightarrow \infty} \langle u, \eta(x, x_0) \rangle + f(x, x_0) > 0$$

for all $x \in K$ and for all $u \in A(x, x)$. Then problem (1) is solvable.

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Generalised Vector Quasi Variational Inequalities

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(Abstract to be given later)

On the existence and upper semicontinuity of solutions to quasivariational inequalities

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Let X and Y be Hausdorff topological vector spaces, U be a Hausdorff topological space and $A \subset X$ be a nonempty, closed and convex subset. Let $C : A \rightarrow 2^Y$, $T : U \rightarrow 2^{L(X,Y)}$ and $K : U \times A \rightarrow 2^X$ be multifunctions with values of C being

closed convex cones with nonempty interiors. Let $g : U \times A \rightarrow A$ be a continuous mapping. Consider the two quasivariational inequality problems with parameters

$$\begin{aligned} \text{(QVI)} : \quad & \text{find } \bar{u} \in U \text{ and } \bar{x} \in A \cap \text{cl}K(\bar{u}, \bar{x}) \text{ such that} \\ & \forall x \in K(\bar{u}, \bar{x}), \exists \bar{t} \in T(\bar{u}, \bar{x}), \\ & (\bar{t}, x - g(\bar{u}, \bar{x})) \in Y \setminus -\text{int}C(\bar{x}), \end{aligned}$$

$$\begin{aligned} \text{(SQVI)} : \quad & \text{find } \bar{u} \in U \text{ and } \bar{x} \in A \cap \text{cl}K(\bar{u}, \bar{x}) \text{ such that} \\ & \forall x \in K(\bar{u}, \bar{x}), \forall t \in T(\bar{u}, \bar{x}), \\ & (t, x - g(\bar{u}, \bar{x})) \in Y \setminus -\text{int}C(\bar{x}). \end{aligned}$$

With relaxed assumptions on semicontinuity and pseudomonotonicity of involved multifunctions (for (QVI) even without pseudomonotonicity) we prove theorems simultaneously on the existence of solutions and its upper semicontinuity with respect to parameter u .

Applications to quasi-complementarity problems and traffic equilibrium problems are also presented. Note that the theorems for (QVI) generalize and improve several recent results in the literature, while the ones for (SQVI) are new, since to our knowledge this problem has never been considered.

Higher - order optimality conditions for isolated local minima

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Let C be a subset of Banach space X , and let S be a closed convex cone of another Banach Space Y . Let f be an extended - real - valued function defined on X , and g be a mapping from X to Y . Consider the optimization problem:

$$\text{(P): } \{ \text{minimize } f(x) \mid -g(x) \in S \text{ and } x \in C \}$$

Higher - order necessary and sufficient optimality conditions for isolated local minima of problem (P) are established in terms of higher - order counterparts of lower and upper Dini directional derivatives.

Hartley Proper Efficiency in Multifunction Optimization

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This paper gives a necessary condition for the Hartley proper efficiency in a vector optimization problem whose objectives and constraints are described by multifunctions F and G . This condition is established under a quasiconvexity requirement of the support function of F and G or a generalized cone-convexity of a multifunction constructed from F and G .

Generalized Derivatives and Generalized Convexity

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(Abstract to be given later)

Convexity of Set-Valued Maps on Set Optimization

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Let (E, \leq) be an ordered topological vector space, and F be a set-valued map from a nonempty set X to E . We consider an optimization problem whose objective map is a set-valued map as follows:

$$\begin{array}{ll} \text{(SP)} & \text{Minimize} \quad F(x) \\ & \text{subject to} \quad x \in X. \end{array}$$

We know that there are two criteria of solutions in these set-valued optimization problems. One is a vector optimization sense (see, for example [3]); x_0 is a solution if $F(x_0)$ has a minimal element of $\text{Min} \bigcup_{x \in X} F(x)$, that is, there exists an element y_0 of $F(x_0)$ such that

$$x \in X, \quad y \in F(x), \quad y \leq y_0 \quad \Rightarrow \quad y_0 \leq y,$$

and the other is a set optimization sense (see, [1]); x_0 is a solution if $F(x_0)$ is a minimal element for some set-relation \preceq , that is,

$$x \in X, \quad F(x) \preceq F(x_0) \quad \Rightarrow \quad F(x_0) \preceq F(x).$$

The former is a usual set-valued optimization problem, and notions of cone-convexity of set-valued maps have been defined and investigated, see [2]. In this paper we consider notions of convexity of set-valued maps in the latter set optimization sense.

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Optimization over the efficient and weakly efficient sets by d.c. programming

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We study concave minimization problems over the efficient and weakly efficient sets of a multi-objective programming problem. They are formulated as d.c. (difference of convex functions) programs in the criteria space, and solved by a d.c. optimization approach called DCA.

Numerical experiments are reported which show the efficiency of the proposed algorithms.

The predictor-corrector DCA for globally solving Large Scale Molecular Optimization from Distance Matrices via reformulations

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In recent years there has been very active research in the molecular optimization, especially in the protein folding framework which is one of the most important problems in biophysical chemistry. Molecular optimization problems arise also in the study of clusters (molecular cluster problems) and of large, confined ionic systems in plasma physics. The determination of a molecular conformation can be tackled by either minimizing a potential energy function (if the molecular structure corresponds to the global minimizer of this function) or solving the distance geometry problem (when the molecular conformation is determined by distances between pairs of atoms in the molecule). Both methods are concerned with global optimization problems.

In this paper we are interested in the large-scale molecular conformation from the distance geometry problem which can be formulated as follows: Find positions of n atoms x^1, \dots, x^n in \mathbb{R}^3 such that

$$\|x^i - x^j\| = \delta_{ij}, \quad (i, j) \in \mathcal{S}, \quad (1)$$

where \mathcal{S} is a subset of the atom pairs, δ_{ij} with $(i, j) \in \mathcal{S}$ is the given distance between atoms i and j , and $\|\cdot\|$ denotes the Euclidean norm. Usually, a small subset of pairwise distances is known, i.e., \mathcal{S} is sparse.

The above formulation corresponds to the exact geometry problem. By the error in the theoretical or experimental data, there may not exist any solution to this problem, for example, when the triangle inequality

$$\delta_{ij} \leq \delta_{ik} + \delta_{kj}$$

is violated for atoms i, j, k . Then an ε -optimal solution of (1), namely a configuration x^1, \dots, x^n satisfying

$$|\|x^i - x^j\| - \delta_{ij}| \leq \varepsilon, \quad (i, j) \in \mathcal{S},$$

is useful in practice.

The general distance geometry problem then is to find positions x^1, \dots, x^n in \mathbb{R}^3 verifying

$$l_{ij} \leq \|x^i - x^j\| \leq u_{ij}, \quad (i, j) \in \mathcal{S},$$

where l_{ij} and u_{ij} are lower and upper bounds of the distance constraints, respectively.

A so-called DCA method based on a d.c. (difference of convex functions) optimization approach for solving large-scale distance geometry problems is developed. Different formulations of equivalent d.c. programs, in the l_1 -approach, are stated via Lagrangian duality without gap relative to d.c. programming and new nonstandard nonsmooth reformulations, in the l_∞ -approach (resp. the $l_1 - l_\infty$ -approach) are introduced. Substantial subdifferential calculations permit to compute quite simply sequences of iterations in the DCA. It actually requires matrix-vector products and only one Cholesky factorization (resp. plus solution of a convex program) in the l_1 -approach (resp. the $l_1 - l_\infty$ -approach), and allows exploiting sparsity in the large-scale setting. Two techniques, using the triangle inequality to generate a complete approximate distance matrix and the spanning trees procedure respectively, were investigated in order to compute a good starting point for the DCA. Finally, many numerical simulations of the molecular optimization problems with up to 12567 variables are reported which prove the practical usefulness of the nonstandard nonsmooth reformulations, the globality of found solutions, the robustness, and the efficiency of our algorithms.

Quality of knowledge Technology, Returns to Production Technology and Economic Development

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Adopting a discrete time version of the Romer (1986) model, this paper analyzes optimal paths in a one-sector growth model when the individual production function is convex-concave and the social production technology exhibits globally increasing returns. We prove that for a given quality of knowledge technology, the countries could take-off if their initial stock of capital are above a critical level. We show that for an economy which wants to take-off by means of knowledge technology requires three factors: large amount of initial knowledge, small fixed costs and a good quality of knowledge technology.

On Connectedness of Solution Sets for Affine Vector Variational Inequality

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The concept of vector variational inequality was introduced by Giannessi in 1980. Since then, many authors have shown that vector variational inequality can be efficient tools for studying multiobjective optimization problems.

In this talk, we will discuss the boundedness and connectedness of solution sets for affine vector variational inequalities with noncompact polyhedral constraint sets and positive semidefinite (or monotone) matrices. Moreover, we show that the boundedness and connectedness results can be applied to multiobjective linear fractional optimization problems and multiobjective convex linear-quadratic optimization problems.

Hidden Convex Minimization

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A class of nonconvex minimization problems can be classified as hidden convex minimization problems. A nonconvex minimization problem is called a hidden convex minimization problem if there exists an equivalent transformation such that the equivalent transformation of it is a convex minimization problem. Sufficient conditions are derived in this paper for identifying such class of seemingly nonconvex minimization problems that are equivalent to convex minimization problems. Thus, a global optimality can be achieved for this class of hidden convex optimization problems by using local search methods. The results presented in this paper extend the reach of convex minimization by identifying its equivalent with a nonconvex representation.

Geometric Properties and Coincidence Theorems with Applications to Generalized Vector Equilibrium Problems

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The present paper is divided into two fold, we derive a Fan-KKM type theorem and establish some geometric properties of convex spaces. By applying our results we also establish some coincidence and fixed point theorems in the setting of convex spaces. Second fold deals with the applications of our coincidence theorems to establish some existence results for a solution to the generalized vector equilibrium problems.

On local uniqueness of solutions of general variational inequalities

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By using Frechet approximate Jacobian matrices, we present some criteria for the local uniqueness of solutions to the general variational inequalities which involve continuous, not necessarily locally Lipschitz continuous data.

On the mix-efficient points

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In this paper the concepts of mix-efficient point and mix quasi-convex set are introduced. By means of these concepts we will investigate the connectedness of the mix-efficient frontier for vector optimization problems defined by quasi-concave, strictly and strong quasi-concave functions. Conditions under which the outcome of a vector function is a set mix-efficient are established. Applications for three criteria problems are given.

Increasing quasiconcave production and utility functions with diminishing returns to scale

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In microeconomic analysis functions with diminishing returns to scale (DRS) have frequently been employed. Various properties of increasing quasiconcave aggregator functions with DRS are derived. Furthermore duality in the classical sense of economic theory as well as a new type of duality are studied for such aggregator functions both in production and consumer theory. In particular, representation theorems for direct and indirect aggregator functions are obtained. These involve only small sets of generator functions. The study is carried out in the contemporary framework of abstract convexity and abstract concavity.

Pseudomonotonicity and Variational Inequality Problems

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Variational inequality problem was introduced by Hartman and Stampacchia while dealing with solutions of partial differential equations. In natural sciences this problem is mostly studied in infinite dimensional spaces. Economists and management scientists are particularly interested in the finite dimensional version. The equilibrium conditions of virtually every equilibrium problem may be formulated as a variational inequality problem. Monotonicity and its generalizations play a crucial role in establishing the solution of variational inequality problem.

The extremal principle and its applications to optimization and economics

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This lecture is devoted to the extremal principle in variational analysis that can be viewed as a variational analogue of the classical convex separation principle

in nonconvex settings. We consider two basic versions of the extremal principle formulated in terms of Fréchet normals and their sequential limits. Then we discuss their relationships with variational principles and their applications to the generalized differential calculus for nonsmooth and set-valued mappings, to necessary optimality and suboptimality conditions, and to the study of Pareto optimality in nonconvex models of welfare economics.

On error bounds for inequality systems in Banach

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(Abstract to be given later)

On Inherited Properties and Scalarization Algorithms for Set-Valued Maps

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This paper consists of two parts which are several inherited properties of set-valued maps and scalarization algorithms for set-valued maps.

Firstly, we present certain results on inherited properties on convexity and semicontinuity. Convexity and lower semicontinuity of real-valued functions are

useful properties for analysis of optimization problems, and they are dual concepts to concavity and upper semicontinuity, respectively. These properties are related to the total ordering of R^n . We consider certain generalizations and modifications of convexity and semicontinuity for set-valued maps in a topological vector space with respect to a cone preorder in the target space, which have studied in [1] for generalizing the classical Fan inequality. These properties are inherited by special scalarizing functions;

$$\inf\{h_C(x, y; k) : y \in F(x)\} \quad (0.1)$$

$$\sup\{h_C(x, y; k) : y \in F(x)\} \quad (0.2)$$

where $h_C(x, y; k) = \inf\{t : y \in tk - C(x)\}$, $C(x)$ is a closed convex cone with nonempty interior, x and y are vectors in two topological vector spaces X, Y , and $k \in \text{int}C(x)$. Note that $h_C(x, \cdot; k)$ is positively homogeneous and subadditive for every fixed $x \in X$ and $k \in \text{int}C(x)$; and moreover $-h_C(x, -y; k) = \sup\{t : y \in tk + C(x)\}$.

Secondly, we develop computational procedures how to calculate the values of functions (0.1) and (0.2). In order to find solutions in multicriteria situations, we use some types of scalarization algorithms such as positive linear functionals and Tchebyshev scalarization. The function $h_C(x, y; k)$ is regarded as a generalization of the Tchebyshev scalarization. By using the function, we give four types of characterizations of set-valued maps.

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Representations of monotone operators by convex functions

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The analogies between closed proper convex functions and maximal monotone operators are striking. Among them are: (a) almost convexity of the domains of such operators, (b) local boundedness on the interiors of their domains, (c) the Brøndsted-Rockafellar theorem, (d) qualification conditions for calculus rules, (e) single valuedness results, (f) regularization processes, etc... These facts prompt one to derive the main results about maximal monotone operators from convex analysis.

Among the possible tools to reach that aim are the representation theorems of Fitzpatrick [9], Krauss [11] - [14], and their supplements by Burachik and Svaiter [7], Martínez-Legaz and Théra [16] which characterize maximal monotone operators. We complete these representations by introducing another natural representation which is closely connected with the restriction of the coupling function c to the graph of the operator. This representation dominates any closed convex function majorized by c on the graph of the operator. This representation and the Fitzpatrick representation are simply related by the Fenchel transformation. We also establish relationships with the Krauss representation which is more complex since it relies on the theory of saddle functions. It has been recently pointed out to the author by B. Svaiter that the new representation of [18] mentioned above is related to the study of enlargements of maximal monotone operators conducted by him and R. Burachik in the forthcoming paper [8] ; see also [4] - [7], [10], [19], [20], [21] . Other representations are given in [17], [22] and in the works by S. Simons.

However, in order to reach our aim, we devise another representation of a maximal monotone operator $M : X \rightrightarrows X^*$, where X is a reflexive Banach space. It is not unique, but it is invariant under the conjugacy obtained by composing the Fenchel conjugacy with a permutation of the variables in $X \times X^*$. In order to prove the existence of such a representation, we use the Zorn lemma, so that our existence result is non constructive. For obtaining calculus rules, we rely on a nice result of R. Burachik and B. Svaiter [7] which characterizes closed convex functions which are Fitzpatrick representations of maximal monotone operators.

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Rough Convexity

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A set M is called roughly convex (in some sense) with respect to a given roughness degree $r > 0$ if certain points of the segment $[x_0, x_1]$ belong to M whenever $x_0, x_1 \in M$ and $\|x_0 - x_1\| \geq r$.

A function $f : M \rightarrow \mathbb{R}$ is said to be roughly convex (in some sense) with respect to a given roughness degree $r > 0$ provided that some convexity property

holds true at certain points of the segment $[x_0, x_1]$ whenever $x_0, x_1 \in M$ and $\|x_0 - x_1\| \geq r$.

We will present some kinds of rough convexity and their application to global optimization.

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Fractional Programming - a recent survey

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Fractional programming is concerned with the optimization of one or several ratios of functions, usually subject to constraints. After about forty years of research well over one thousand articles have appeared, in addition to the monographs by Schaible (1978), Craven (1988) and Stancu-Minasian (1997) dealing with applications, theory and solution methods.

The purpose of this survey is to identify some recent developments in fractional programming. To make the survey somewhat self-contained we provide briefly the necessary background from the known literature.

We consider single-ratio as well as multi-ratio fractional programs. In the latter case, we focus on the maximization of the smallest of several ratios and the maximization of a sum of ratios with an emphasis on the difficult sum-of-ratios fractional problem.

Some recent results on (nonconvex) quadratic programming

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(Abstract to be given later)

Equilibrium in an exchange economy and quasiconvex duality

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(Abstract to be given later)

Convergence of duality bound methods for programming problems dealing with partly convex functions

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We discuss the convergence of a decomposition branch and bound algorithm using Lagrangian duality for partly convex programs in the general form. It is shown that this decomposition algorithm has all useful convergence properties for solving the underlying problem class under usual assumptions. Thus, some strict assumptions discussed in the literature are avoidable.

Monotonicity in a Framework of Generalized Convexity

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Given a family \mathcal{L} of real-valued functions on R^n a set $G \subset R^n$ is said to be convex w.r.t. \mathcal{L} or \mathcal{L} -convex if for any point $a \in R^n \setminus G$ there exists a function $\ell \in \mathcal{L}$ separating a from G , i.e. such that $\ell(a) \geq 0$ and $\ell(x) \leq 0 \forall x \in G$. A function $f : R^n \rightarrow R \cup \{+\infty\}$ is said to be \mathcal{L} -convex if for any $\alpha \in R$ the set $\{x \in R^n \mid f(x) \leq \alpha\}$ is \mathcal{L} -convex.

An increasing function $f : R^n \rightarrow R \cup \{+\infty\}$ is a function such that $f(x') \geq f(x)$ whenever $x' \geq x$ (i.e. $x'_i \geq x_i \forall i$). A downward set $G \subset R^n$ is a set such that $x' \in G$ whenever $x' \leq x$ for some $x \in G$. We present a geometric theory of monotonicity in which increasing functions and downward sets are convex functions and convex sets w.r.t. the family \mathcal{L} of functions of the form $\ell(x) = \min_i \{x_i - a_i\}$, $a \in R^n$. In particular, the concepts of \mathcal{L} -convex hull (downward hull), \mathcal{L} -polytope (polyblock) and extreme point are introduced such that several properties hold that remind similar facts from convex analysis : any closed \mathcal{L} -convex (downward) set is the intersection of a family of \mathcal{L} -polytopes, any closed \mathcal{L} -convex set is the \mathcal{L} -convex hull of the set of its extreme points, a \mathcal{L} -polytope is the \mathcal{L} -hull of a finite set, the set of extreme points of the \mathcal{L} -convex hull of a compact set K is a subset of K , etc.

We also discuss applications to the study of systems of monotonic inequalities and optimization problems of the form: $\max\{f(x) \mid g(x) \leq 0 \leq h(x)\}$ where f, g, h are increasing.

Generalized distance and its applications

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(Abstract to be given later)

Duality methods via augmented Lagrangian functions

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There are a number of augmented Lagrangian schemes in the literature, which provide the zero duality gap and the exact penalty property. In this talk, a generalized augmented Lagrangian scheme is discussed. Under weaker conditions,

this new scheme is also able to provide the zero duality gap and the exact penalty property. Moreover, the equivalences among the zero duality gap results which are obtained using various duality functions are reviewed.

Mordukhovich's Coderivative for Multifunctions and Implicit Function Theorems

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In the first part of this talk, I would like to present some remarks about the history and the role of B. Mordukhovich's theory of coderivative for multifunctions.

In the second part of this talk, I shall outline a way to obtain new implicit function theorems for set-valued maps by using the above-mentioned theory of coderivative.

Generalized equilibrium for quasimonotone and pseudomonotone bifunctions

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By using quasimonotone and pseudomonotone bifunctions, we derive sufficient conditions which include weak coercivity conditions for existence of equilibrium points. As consequences we generalize various recent results on the existence of such solutions and for variational inequalities.

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