Lecture Notes in Economics and Mathematical Systems 616

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Generalized Convexity and Optimization

Theory and Applications

Springer

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ISBN 978-3-540-70875-9 e-ISBN 978-3-540-70876-6

DOI 10.1007/978-3-540-70876-6

Lecture Notes in Economics and Mathematical Systems ISSN 0075-8442

Library of Congress Control Number: 2008933853

Cover design: WMXDesign GmbH, Heidelberg

Printed on acid-free paper

9 8 7 6 5 4 3 2 1

springer.com

To Giovanna and Paolo

Foreword

In the latter part of the twentieth century, the topic of generalizations of convex functions has attracted a sizable number of researchers, both in mathematics and in professional disciplines such as economics/management and engineering. In 1994 during the 15th International Symposium on Mathematical Programming in Ann Arbor, Michigan, I called together some colleagues to start an affiliation of researchers working in generalized convexity. The international Working Group of Generalized Convexity (WGGC) was born. Its website at www.genconv.org has been maintained by Riccardo Cambini, University of Pisa.

Riccardo's father, Alberto Cambini, and Alberto's long-term colleague Laura Martein in the Faculty of Economics, University of Pisa, are the coauthors of this volume. My own contact with generalized convexity in Italy dates back to my first visit to their department in 1980, at a time when the first international conference on generalized convexity was in preparation. Thirty years later it is now referred to as GC1, an NATO Summer School in Vancouver, Canada. Currently WGGC is preparing GC9 which is to take place in Kaohsiung, Taiwan. As founding chair and also current chair of WGGC, I am delighted to see the continued interest in generalized convexity of functions, augmented by the topic of generalized monotonicity of maps.

Eight international conferences have taken place in this research area, in North America (2), Europe (5) and Asia (1). We thought it was now time to return to Asia since our membership has shifted towards Asia.

As an applied mathematician I have taught mostly in management schools. However, I am currently in the process of joining an applied mathematics department. One of the first texts I will try out with my mathematics students is this volume of my long-term friends from Pisa. I recommend this volume to anyone who is trying to teach generalized convexity/generalized monotonicity in an applied mathematics department or in a professional school. The volume is suitable as a text for both. It contains proofs and exercises. It also provides sufficient references for those who want to dig deeper as graduate students and as researchers. With dedication and much love the authors have written a book that is useful for anyone with a limited background in basic mathematics. At the same time, it also leads to more advanced mathematics.

The classical concepts of generalized convexity are introduced in Chaps. 2 and 3 with separate sections on non-differentiable and differentiable functions. This has not been done in earlier presentations. Chapter 4 deals with the relationship of optimality conditions and generalized convexity. One of the reasons for a study of generalized convexity is that convexity usually is just a convenient sufficient condition. In fact most of the time it is not necessary. And it is a rather rigid assumption, often not satisfied in real-world applications. That is the reason why economists have replaced it by weaker assumptions in more contemporary studies. In fact, some of the progress in this research area is due to the work of economists. I am glad that the new book emphasizes economic applications.

In Chap. 5 the transition from generalized convexity to generalized monotonicity occurs. Historically, this happened only around 1990 when I was working with the late Stepan Karamardian after joining the University of California at Riverside. He was a former PhD student of George Dantzig at the University of California at Berkeley. We collaborated on the last two papers he published, both on generalized monotonicity. (a new research area) We had opened up together.

In 2005 Nicolas Hadjisavvas, Sandor Komlosi and I completed the first *Handbook of Generalized Convexity and Generalized Monotonicity* with contributions from many leading experts in the field, including Alberto Cambini and Laura Martein, a proven team of co-authors who in their unique colorful way have left an imprint in the field. The new book is further evidence of their style.

Chapters 6 and 7 are devoted to specialized results for quadratic functions and fractional functions. With this the authors follow the outline of the first monograph in this research area, *Generalized Concavity* by Mordecai Avriel, Walter E. Diewert, Siegfried Schaible and Israel Zang in 1988. Chapter 8 contains algorithmic material on solving generalized convex fractional programs. It defeats the objection sometimes raised that the area of generalized convexity lacks algorithmic contributions. It is true that there could be more results in this important direction on a topic which by nature is theoretical. Perhaps the presentation in Chap. 8 will motivate others to take up the challenge to derive more results with a computational emphasis.

Today *Generalized Concavity* (1988) is available to us as the first volume on the topic, together with the comprehensive *Handbook of Generalized Convexity and Generalized Monotonicity* (2005), an edited volume of 672 pages, written by 16 different researchers including Alberto Cambini and Laura Martein. In addition, the published proceedings of GC1–GC8 are available from reputable publishing houses. The proceedings of GC9 will appear partially in the prestigious Taiwanese Journal of Mathematics.

As somebody who has participated in all the conferences, GC1–GC8, and who is co-organizing GC9 together with Jen-Chih Yao, Kaohsiung and who has been involved in most publications mentioned before, I congratulate the authors for having produced such a fine volume in this growing area of research. Like me they stumbled into it when no monographs on the topic were available. I can see the usefulness of the book for teaching and research for generations to come. Its technical level makes it suitable for undergraduate and graduate students. The level is pitched wisely. The book is more accessible than the Handbook as it assumes less background knowledge about the topic. This is not surprising as the purpose of the Handbook is different. The new book can serve as an up-to-date link to the Handbook. It also saves the reader from going through the earlier proceedings with more dated results.

As someone who, like the authors, has not departed from the area of generalized convexity in his career, I can highly recommend this excellent new volume in our community of researchers. WGGC has been the background for most recent publications in our field of study. It is the excitement of working in teams which has been promoted by WGGC. A sense of community very common in Italy is the background of this new volume. It made me happy when I reviewed the manuscript first. I hope that many readers will come to the same conclusion. My thanks and congratulations go to the authors for a job well done.

I want to thank the authors for having taken the time to write *Generalized Convexity and Optimization with Economic Applications* and for their diligent effort to produce an up-to-date text and wish the book much success among our growing community of researchers.

Riverside, California,
June 2008

Siegfried Schaible
Chair of WGGC

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