# Nonconvex Optimization and Its Applications

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# Invexity and Optimization

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The first author dedicates this book to his son Rohan Mishra
The second author dedicates this book to his mother Olga

# Preface

Generalized convexity and generalized monotonicity are the core of many important subjects in the context of various research fields such as mathematics, economics, management science, engineering and other applied sciences. After the introduction of quasi-convex functions by de Finetti in 1949, many other authors have defined and studied several types of generalized convex functions and their applications in the context of scalar and vector optimization problems, calculus of variations and optimal control theory and financial and economic decision models.

In many cases, generalized convex functions preserve some of the valuable properties of convex functions. One of the important generalizations of convex functions is invex functions, a notion originally introduced for differentiable functions $f : X \to R$, $X$ an open set of $R^n$, for which there exists some function $\eta : X \times X \to R^n$ such that $f(x) - f(y) \geq \eta(x, y)^T \nabla f(u), \forall x, u \in X$. Invex functions have the property that all stationary points are global minimizers and, since their introduction in 1981, have been used in many applications.

The interest in these topics is continuous, as shown by eight specific international meetings (the next one is scheduled in Kaohsiung, July 21–25, 2008, along with 2nd Summer School for Generalized convexity from July 15–19, 2008 at the Department of Applied Mathematics, The National Sun Yat-sen University, Kaohsiung, Taiwan) held to date (Vancouver in 1980; Canton (USA) in 1986; Pisa in 1988; Pecs (Hungary) in 1992; Luminy (France) in 1996; Hanoi (Vietnam) in 2001; Varese (Italy) in 2005) and by the foundation of the Scientific Committee of the Working Group on Generalized Convexity, the group sponsored by the Mathematical Programming Society.

This book deals with invex functions and their applications in nonlinear scalar and vector optimization problems, nonsmooth optimization problems, fractional and quadratic programming problems and continuous-time optimization problems. This book provides a comprehensive discussion on invex functions and their applications, based on the research work carried out over the past several decades.

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# Contents