

Nonconvex Optimization and Its Applications

Volume 90

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Generalized Convexity and Vector Optimization

 Springer

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ISBN 978-3-540-85670-2 e-ISBN 978-3-540-85671-9

Nonconvex Optimization and Its Applications ISSN 1571-568X

Library of Congress Control Number: 2008936488

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Cover design: WMX Design GmbH, Heidelberg

Printed on acid-free paper

9 8 7 6 5 4 3 2 1

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Preface

The present lecture note is dedicated to the study of the optimality conditions and the duality results for nonlinear vector optimization problems, in finite and infinite dimensions. The problems include are nonlinear vector optimization problems, symmetric dual problems, continuous-time vector optimization problems, relationships between vector optimization and variational inequality problems.

Nonlinear vector optimization problems arise in several contexts such as in the building and interpretation of economic models; the study of various technological processes; the development of optimal choices in finance; management science; production processes; transportation problems and statistical decisions, etc.

In preparing this lecture note a special effort has been made to obtain a self-contained treatment of the subjects; so we hope that this may be a suitable source for a beginner in this fast growing area of research, a semester graduate course in nonlinear programming, and a good reference book. This book may be useful to theoretical economists, engineers, and applied researchers involved in this area of active research.

The lecture note is divided into eight chapters:

Chapter 1 briefly deals with the notion of nonlinear programming problems with basic notations and preliminaries.

Chapter 2 deals with various concepts of convex sets, convex functions, invex set, invex functions, quasiinvex functions, pseudoinvex functions, type I and generalized type I functions, V-invex functions, and univex functions.

Chapter 3 covers some new type of generalized convex functions, such as Type I univex functions, generalized type I univex functions, nondifferentiable d-type I, nondifferentiable pseudo-d-type I, nondifferentiable quasi d-type I and related functions, and similar concepts for continuous-time case, for nonsmooth continuous-time case, and for n-set functions are introduced.

Chapter 4 deals with the optimality conditions for multiobjective programming problems, nondifferentiable programming problems, minimax fractional programming problems, mathematical programming problems in Banach spaces, in complex spaces, continuous-time programming problems, nonsmooth continuous-time programming

problems, and multiobjective fractional subset programming problems under the assumptions of some generalized convexity given in Chap. 3.

In Chap. 5 we give Mond–Weir type and General Mond–Weir type duality results for primal problems given in Chap. 4. Moreover, duality results for nonsmooth programming problems and control problems are also given in Chap. 5.

Chapter 6 deals with second and higher order duality results for minimax programming problems, nondifferentiable minimax programming problems, nondifferentiable mathematical programming problems under assumptions generalized convexity conditions.

Chapter 7 is about symmetric duality results for mathematical programming problems, mixed symmetric duality results for nondifferentiable multiobjective programming problems, minimax mixed integer programming problems, and symmetric duality results for nondifferentiable multiobjective fractional variational problems.

Chapter 8 is about relationships between vector variational-like inequality problems and vector optimization problems under various assumptions of generalized convexity. Such relationships are also studied for nonsmooth vector optimization problems as well. Some characterization of generalized univex functions using generalized monotonicity are also given in this chapter.

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