Generalized Concavity in Fuzzy Optimization and Decision Analysis

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GENERALIZED CONCAVITY IN FUZZY OPTIMIZATION AND DECISION ANALYSIS

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Contents

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List of Symbols

Symbol	*Description*
$\mathbf{R}^n$	n-dimensional (Euclidean) real vector space
$f : X \to Y$	mapping or function f that maps a set X into a set Y
$\mathrm{Ran}(f)$	range of f
$f^{(-1)}$	pseudo-inverse function to f
$\langle x, y\rangle$	inner product of x and y
$\|\|x\|\|$	norm of x
$d(x, y)$	distance between x and y
$B(x, \delta)$	open ball with center x and radius δ
$[0, 1]$	unit interval in $\mathbf{R}$
$\mathcal{C}(S)$	complement of S
$\mathrm{Ker}(X)$	kernel of X
$\mathrm{Ker}^*(X)$	strong kernel of X
$\mathrm{Ker}_\infty(X)$	co-kernel of X
$\mathrm{Ker}^*_\infty(X)$	strong co-kernel of X
$\mathrm{Int}(S)$	interior of S
$\mathrm{Rlint}(S)$	relative interior of S
$\mathrm{Cl}(S)$	closure of S
$\mathrm{Bd}(S)$	boundary of S
$U(f, \alpha)$	upper level set of f at α
$L(f, \alpha)$	lower level set of f at α
$H(f, \alpha)$	level set of f at α
$\mathrm{Epi}(f)$	epigraph of f
$\mathrm{Hyp}(f)$	hypograph of f
$\mathbf{I}(x, y)$	line segment joining x and y
$\mathbf{L}(x, y)$	line going through x and y
$\mathbf{H}(x, y)$	half line emanating from x through y
$\mathrm{Conv}(S)$	convex hull of S
$\dim(S)$	dimension of S
$\mathrm{Card}(S)$	cardinality of S, number of elements of S
$\mathrm{Ext}(S)$	set of all extreme points of S
$\mathrm{Core}(\mu)$	core of μ
$\mathrm{Supp}(\mu)$	support of μ
$\nabla f(x)$	gradient vector of f at x
$\nabla^2 f(x)$	Hessian matrix of f at x
T_M, S_M	minimum t-norm, maximum t-conorm
T_P, S_P	product t-norm, probabilistic sum t-conorm
T_L, S_L	Łukasiewicz t-norm, bounded sum t-conorm
T_D, S_D	drastic product t-norm, drastic sum t-conorm
OS^k	k-order statistic aggregation operator
OWA^n_W	order weighted averaging operator of dimension n
$[A]_\alpha$	α-cut of a fuzzy set A
$\mathcal{F}(X)$	set of all fuzzy subsets of X
$\mathcal{C}_N A$	complement of fuzzy set A with respect to negation N
$\mu_{\tilde{R}T}(A, B)$	T-fuzzy extension of relation R of fuzzy sets A and B

Preface

A large number of decision making and optimization problems can be formulated as follows: Given a set of *feasible alternatives* and a binary relation *better than* for a consistent mutual comparison of alternatives, find the *best* alternative. As a rule, the set of feasible alternatives is specified by a number of conditions as a subset of a given underlying set. The underlying set is usually equipped with some mathematical structure that can be more or less helpful in searching for the best feasible alternative. For almost all parts of this book, the underlying set is a finite-dimensional Euclidean space.

In a typical deterministic framework, the binary relation enabling comparison of alternatives is represented by a real-valued function f mapping the set of feasible alternatives into the set of real numbers in such a manner that a feasible alternative x is better than a feasible alternative y if and only if $f(x) > f(y)$ or $f(x) < f(y)$, one of these possibilities selected. In the former case the problem of finding the best alternative becomes that of maximizing f over the set of feasible alternatives; in the latter case, minimization of f is required. Construction of such an objective function may be a nontrivial task. Moreover, for some practically relevant binary relations such representations do not exist. It is therefore sometimes preferable or necessary to represent some relations by means of several functions. The meaning of maximization or minimization with respect of several real-valued functions may vary according to the underlying binary relation. For example: in some situations, the decision maker can be interested in finding a Pareto maximizer; in other situations, his wish may be to find a compromise solution.

Convexity of sets in linear spaces, and concavity and convexity of functions lie at the root of beautiful theoretical results that are at the same time extremely useful in the analysis and solution optimization problems, regardless of whether the optimization is required with respect to a single objective or multiple objectives. Fortunately, not each of these results relies necessarily on convexity or concavity. Some of them, for example the results guaranteeing that each local optimum is also a global optimum, can be derived for substantially wider classes of problems. A large portion of the first part of this book is concerned with several types of generalized convex sets and generalized concave functions. In addition to their applicability to nonconvex optimization, they are used in the second part, where decision making and optimization problems under uncertainty are investigated.

Uncertainty in the problem data often cannot be avoided when dealing with practical problems. It may arise from errors in measuring physical quantities, from errors caused by representing some data in a computer, from the fact that some data are approximate solutions of other problems or estimations by human experts, etc. Over the last thirty years, the fuzzy set approach proved to be useful in some of these situations. It is this approach to optimization under uncertainty that is extensively used and studied in the second part of this book.

Usually, the membership functions of fuzzy sets involved in such problems are neither concave nor convex. They are, however, often quasiconcave or concave in some generalized sense. This opens possibilities for application of results on generalized concavity to fuzzy optimization. Interestingly, despite of this obvious relation, the interaction between these two areas has been rather limited so far. It is hoped that the presented combination of ideas and results from the field of generalized concavity on the one hand and fuzzy optimization on the other hand will be of interest to both communities and will result in an enlargement of the class of problems that can be satisfactorily handled.

In Chapter 1, for reader's convenience, we review some basic notation and concepts necessary for understanding of the text, particularly, we present some introductory elements of linear algebra and calculus.

In Chapter 2, we deal with generalized convex sets. The most natural generalization of convex sets are starshaped sets. As further generalization of starshaped sets we obtain path-connected sets, invex sets and univex sets. Finally, we introduce a new class of generalized convex sets.

Chapter 3 begins with the classical definitions of concave and quasiconcave functions. Then we introduce starshaped functions, quasiconnected functions, and a concavity of functions with respect to suitable sets of mappings and functions. This approach enable us to cover several other ways of introducing generalized concavity known from the literature. Differentiable generalized concave functions are also studied and mutual relationships among different classes of functions are presented. An application to constrained optimization is discussed.

In Chapter 4, we deal with functions defined on the n-dimensional Euclidean space $\mathbf{R}^n$ and having their values in the unit interval $[0, 1]$ of real numbers. Such functions naturally appear in optimization under uncertainty where they can be interpreted as membership functions of fuzzy subsets of $\mathbf{R}^n$ or possibility distributions, etc. Using the notions and properties of triangular norms and triangular conorms, we introduce and study new classes of functions related to concavity. We call them (Φ, T)-concave functions because they are defined by choosing a class Φ of suitable mappings and a triangular norm T. We focus on deriving conditions under which local maximizers of such functions are also global maximizers.

In Chapter 5, we prepare a material for further study of multi-objective optimization problems by analyzing some properties of general aggregation operators applied to criteria in the form of generalized concave functions. We look for conditions guaranteeing some attractive local-global properties of aggregating mappings. In this context we review mainly well known classes of averaging aggregation operators: compensative operators, order-statistic operators and OWA operators. We present also general classes of aggregation operators generated by Sugeno and Choquet integrals.

In Chapter 6, we deal with fuzzy sets. Already in the early stages of the development of fuzzy set theory, it has been recognized that fuzzy sets can be defined and represented in several different ways. We define fuzzy sets within the classical set theory by nested families of sets, and then we discuss how this concept is related to the usual definition by membership functions. Binary and valued relations are extended to fuzzy relations and their properties are extensively investigated. Moreover, fuzzy extensions of real functions are studied, particularly, the problem of establishing sufficient conditions under which the membership function of the function value is quasiconcave. Sufficient conditions for commuting the diagram "mapping - α-cutting" is presented in the form of classical Nguyen's result.

In the second part of this book, we are concerned with applications of the theory presented in the first part.

In Chapter 7, we consider the problem to find a "best" decision in the set of feasible decisions with respect to several criteria functions. Within the framework of such a decision situation, we deal with the existence and mutual relationships of three kinds of "optimal decisions": Weak Pareto-Maximizers, Pareto-Maximizers and Strong Pareto-Maximizers - particular alternatives satisfying some natural and rational conditions. We study also the compromise decisions maximizing some aggregation of the criteria. The criteria considered will be functions defined on the set of feasible decisions with the values in the unit interval. As mentioned above such functions can be interpreted as membership functions of fuzzy subsets and will be called fuzzy criteria. Later on, in Chapters 8 and 9, each constraint or objective function of the fuzzy mathematical programming problem will be associated with a unique fuzzy criterion. From this point of view, Chapter 7 could follow the Chapters 8 and 9, which deal with fuzzy mathematical programming. Our approach is, however, more general and can be adopted to a more general class of decision problems. The results of Chapter 5 are extended and presented in the framework of multicriteria decision making.

Fuzzy mathematical programming problems (FMP) investigated in Chapter 8 form a subclass of decision-making problems where preferences between alternatives are described by means of objective functions defined on the set of alternatives in such a way that greater values of the functions correspond to more preferable alternatives. The values of the objective function describe effects from choices of the alternatives. The chapter begins with the formulation of a FMP problem associated with the classical mathematical programming problem (MP). After that we define a feasible solution of FMP problem and optimal solution of FMP problem as special fuzzy sets. From practical point of view, α-cuts of these fuzzy sets are important, particularly the nonempty α-cuts with the maximal α. Among others we show that the class of all MP problems with (crisp) parameters can be naturally embedded into the class of FMP problems with fuzzy parameters.

In Chapter 9, we deal with a class of fuzzy linear programming problems (FLP) and again introduce the concepts of feasible and optimal solutions - the necessary tools for dealing with such problems. In this way we show that the class of classical linear programming problems (LP) can be embedded into the class of FLP problems. Moreover, for FLP problems we define the concept of duality and prove the weak and strong duality theorems. Furthermore, we investigate special classes of FLP - interval LP problems, flexible LP problems, LP problems with interactive coefficients and LP problems with centered coefficients.

In Chapter 10, we first recall elementary concepts and basic models of deterministic machine scheduling. Then we discuss some of them in nondeterministic situations. We present motivation examples characterizing difficulties that may occur under uncertainty of problem parameters. Then we investigate some particular fuzzy models with fuzzy due dates, fuzzy processing times and fuzzy precedence relations. Finally we discuss some directions of the future research in the area of fuzzy machine scheduling.

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A famous theorem says that there is an error or misprint in every mathematical text longer than n pages where n is a small natural number. We have tried to minimize them but as no theorem can be false, we will be grateful for receiving information on errors, as well as comments and suggestions for improvement.