

# **Infosys Science Foundation Series**

## **Infosys Science Foundation Series in Mathematical Sciences**

### **Series editors**

Gopal Prasad, University of Michigan, USA  
Irene Fonseca, Mellon College of Science, USA

### **Editorial Board**

Chandrasekhar Khare, University of California, USA  
Mahan Mj, Tata Institute of Fundamental Research, Mumbai, India  
Manindra Agrawal, Indian Institute of Technology Kanpur, India  
S.R.S. Varadhan, Courant Institute of Mathematical Sciences, USA  
Weinan E, Princeton University, USA

The *Infosys Science Foundation Series in Mathematical Sciences* is a sub-series of The *Infosys Science Foundation Series*. This sub-series focuses on high quality content in the domain of mathematical sciences and various disciplines of mathematics, statistics, bio-mathematics, financial mathematics, applied mathematics, operations research, applied statistics and computer science. All content published in the sub-series are written, edited, or vetted by the laureates or jury members of the Infosys Prize. With the Series, Springer and the Infosys Science Foundation hope to provide readers with monographs, handbooks, professional books and textbooks of the highest academic quality on current topics in relevant disciplines. Literature in this sub-series will appeal to a wide audience of researchers, students, educators, and professionals across mathematics, applied mathematics, statistics and computer science disciplines.

More information about this series at <http://www.springer.com/series/13817>

Ram U. Verma

# Semi-Infinite Fractional Programming

 Springer

Ram U. Verma  
Department of Mathematics  
Texas State University  
San Marcos, TX  
USA

ISSN 2363-6149                      ISSN 2363-6157 (electronic)  
Infosys Science Foundation Series  
ISSN 2364-4036                      ISSN 2364-4044 (electronic)  
Infosys Science Foundation Series in Mathematical Sciences  
ISBN 978-981-10-6255-1            ISBN 978-981-10-6256-8 (eBook)  
DOI 10.1007/978-981-10-6256-8

Library of Congress Control Number: 2017951997

© Springer Nature Singapore Pte Ltd. 2017

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Printed on acid-free paper

This Springer imprint is published by Springer Nature

The registered company is Springer Nature Singapore Pte Ltd.

The registered company address is: 152 Beach Road, #21-01/04 Gateway East, Singapore 189721, Singapore

# Preface

This monograph is aimed at presenting a smooth and unified transition from the general fractional programming (or program) to the semi-infinite fractional programming (or program), especially in terms of framework for theoretical foundation and real-world applications, some new classes of generalized second-order invex functions and second-order univex functions, new sets of second-order necessary optimality conditions, numerous sets of second-order sufficient optimality conditions, and several sets of second-order duality models for establishing numerous duality theorems for a discrete minmax (or maxmin) semi-infinite fractional programming problem. Under the current interdisciplinary computer-oriented research environment, semi-infinite fractional programming is among the most rapidly expanding research areas in terms of its multifacet applications empowerment for real-world problems that can be handled by transforming them into semi-infinite fractional programming problems.

A mathematical fractional programming problem with a finite number of variables and infinitely many constraints is called a semi-infinite fractional programming problem in the literature. On the other hand, mathematical fractional programming problems with a finite number of constraints are referred to as generalized fractional programming problems, and they have been the focus of more research endeavors in terms of providing realistic models for some significant real-world problems, including the problems encountered in multiobjective programming, approximation theory, goal programming, facility location planning, and economics, and their mathematical tractability (they can be transformed into equivalent parametric nonlinear programming problems with nonfractional objective functions).

As a matter of fact, in mathematical optimization programs, a fractional programming (or program) is a generalization to linear fractional programming. These problems lay the theoretical foundation that will enable us to fully investigate the second-order optimality and duality aspects of our principal fractional programming problem as well as the semi-infinite counterpart, which is the main focus of this monograph to empower graduate students, faculty, and other research enthusiasts for more accelerated research advances with significant applications in the

interdisciplinary sense without borders. The main ingredients of this presentation are as follows:

Parametric Optimality  
 Parametric Duality  
 Parameter-free Optimality  
 Parameter-free Duality  
 Nonfractional Optimality  
 Nonfractional Duality

There are some quality books and surveys on generalized fractional programming problems, while it seems there are no such references exclusively in semi-infinite fractional programming other than some surveys in the literature since semi-infinite fractional programming is a new fast-developing research field transitioning from the generalized fractional programming. Furthermore, the methodology (the use of alternative concepts, partitioning schemes, and duality models) adopted in this book, as well as for the main results, will prove useful for other classes of nonlinear semi-infinite fractional programming and beyond. The generalized fractional programming problems have a wide range of real-world problems, which can be transformed into some sort of a generalized fractional programming problem.

Let us consider fractional programs that arise from management decision science. If we consider a system efficiency in an economical sense, it is equivalent to maximizing system efficiency leading to fractional programs with occurring objectives:

Maximizing productivity  
 Maximizing return on investment  
 Maximizing return/risk  
 Minimizing cost/time  
 Minimizing output/input

We envision that this monograph is a unique presentation of interdisciplinary research for the world scientific community (including graduate students, faculty, and general readers). Furthermore, some of the new concepts can be applied to duality theorems based on using a new class of multitime multiobjective variational problems as well.

I am extremely grateful to Prof. G.J. Zalmi, Northern Michigan University, for giving me tremendous opportunities for collaborative research, especially relating to semi-infinite fractional programming problems in a series of research publications, while most of the new concepts are already referred to as Zalmi-type sonvexities and Zalmi-type sounvexities in the literature. Furthermore, I express my deepest appreciation to Prof. R.N. Mohapatra, University of Central Florida, for his guiding star roles during my academic adventures and beyond.

San Marcos, USA  
 February 2017

Ram U. Verma

# Contents

<b>1</b>	<b>Higher Order Parametric Duality Models</b> . . . . .	1
1	Role of Parametric Duality Models . . . . .	1
2	Generalized Sonvexities . . . . .	2
3	New Duality Models . . . . .	6
4	Duality Theorems . . . . .	8
5	General Remarks . . . . .	15
	References . . . . .	15
<b>2</b>	<b>New Generation Parametric Optimality</b> . . . . .	17
1	The Significance of Semi-infinite Fractional Programming . . . . .	17
2	Basic Concepts and Auxiliary Results . . . . .	18
3	Sufficient Optimality Theorems . . . . .	21
4	General Remarks . . . . .	34
	References . . . . .	36
<b>3</b>	<b>Accelerated Roles for Parametric Optimality</b> . . . . .	37
1	Semi-infinite Fractional Programming . . . . .	37
2	General Concepts and Auxiliary Results . . . . .	38
3	New Sufficient Optimality Conditions . . . . .	41
4	General Remarks . . . . .	55
	References . . . . .	58
<b>4</b>	<b>Semi-infinite Multiobjective Fractional Programming I</b> . . . . .	59
1	Role of Sufficient Efficiency Conditions . . . . .	59
2	Basic Concepts . . . . .	61
3	Sufficient Efficiency Theorems . . . . .	66
4	General Remarks . . . . .	81
	References . . . . .	82

<b>5</b>	<b>Semi-infinite Multiobjective Fractional Programming II</b> . . . . .	83
1	Hadamard Derivatives and Parametric Duality Models . . . . .	83
2	Significant Basic Concepts . . . . .	85
3	Duality Model I . . . . .	87
4	Duality Model II . . . . .	94
5	Some Applications . . . . .	111
6	General Remarks . . . . .	113
	References . . . . .	114
<b>6</b>	<b>Semi-infinite Multiobjective Fractional Programming III</b> . . . . .	115
1	Role of Semi-infinite Multiobjective Fractional Programs . . . . .	115
2	Significant Basic Concepts and Auxiliary Results . . . . .	116
3	Duality Model I and Duality Theorems . . . . .	119
4	Duality Model II and Duality Theorems . . . . .	126
5	Some Applications . . . . .	146
6	General Remarks . . . . .	149
	References . . . . .	150
<b>7</b>	<b>Hanson-Antczak-Type Generalized V-Invexity I</b> . . . . .	151
1	Role of Sufficient Conditions . . . . .	151
2	Hanson-Antczak-Type Invexities . . . . .	153
3	Sufficient Efficiency Conditions . . . . .	158
4	Generalized Sufficiency Criteria . . . . .	165
5	General Remarks . . . . .	171
	Reference . . . . .	172
<b>8</b>	<b>Parametric Optimality in Semi-infinite Fractional Programs</b> . . . . .	173
1	Role of Optimality in Semi-infinite Fractional Programming . . . . .	173
2	Generalized Sonvexities . . . . .	174
3	Parametric Necessary and Sufficient Optimality . . . . .	176
4	General Remarks . . . . .	190
	References . . . . .	190
<b>9</b>	<b>Semi-infinite Discrete Minmax Fractional Programs</b> . . . . .	193
1	Significance of Semi-infinite Fractional Programming . . . . .	193
2	Hybrid Sonvexities . . . . .	194
3	Main Results on Necessary and Sufficient Optimality . . . . .	196
4	General Remarks . . . . .	209
	References . . . . .	209
<b>10</b>	<b>Next-Generation Semi-infinite Discrete Fractional Programs</b> . . . . .	211
1	Necessary and Sufficient Optimality . . . . .	211
2	Generalized Sonvexities . . . . .	212
3	Necessary and Sufficient Optimality Theorems . . . . .	213
4	General Remarks . . . . .	230
	References . . . . .	230

- 11 Hanson-Antczak-Type Sonvexity III** . . . . . 231
  - 1 Semi-infinite Multiobjective Fractional Programming . . . . . 231
  - 2 Hanson-Antczak Type Sonvexities . . . . . 234
  - 3 Duality Models I . . . . . 239
  - 4 Duality Model II . . . . . 243
  - 5 Duality Model III . . . . . 249
  - 6 Duality Model IV . . . . . 256
  - 7 General Remarks . . . . . 263
  - References . . . . . 263
- 12 Semi-infinite Multiobjective Optimization** . . . . . 265
  - 1 The Significance of Semi-infinite Multiobjective Optimization . . . 265
  - 2 Significant Related Concepts . . . . . 266
  - 3 A Theorem of the Alternative and Necessary Efficiency Conditions . . . . . 268
  - 4 Role of Hadamard Differentiability . . . . . 271
  - 5 Role of Gâteaux Differentiability . . . . . 274
  - 6 Some Applications . . . . . 275
  - 7 Significant Specializations . . . . . 279
  - 8 General Remarks . . . . . 281
  - References . . . . . 281
- References** . . . . . 283

## About the Author

**Ram U. Verma** is President of International Publications, USA. Before joining Texas State University, he held several academic positions, ranging from Lecturer to Assistant Professor, Associate Professor, and Full Professor at the University of Cape Coast, the University of Tripoli, the University of Orient, the University of Puerto Rico, New York University (visiting faculty), the University of Central Florida, Mount Olive College, Duke University (visiting scholar), and the University of Toledo. His research interests encompass mathematical programming, fractional programming, semi-infinite fractional programming, multiobjective fractional programming, numerical analysis, generalized Newton's methods, new generation Newton-type methods, nonlinear functional analysis, applied analysis, evolution equations and semigroups, stochastic analysis, mathematics education, and determinant theory for singular integral equations.

He has published over 700 research articles in several international refereed journals including *Applied Mathematics and Computation*, *Applicable Analysis*, *Archivum Math*, *Communications in Nonlinear Science and Numerical Simulations*, *Czechoslovak Mathematical Journal*, *Electron*, *Journal of Differential Equations*, *Journal of Computational Analysis and Applications*, *Journal of Mathematical Analysis and Applications*, *Journal of Optimization Theory and Applications*, *Nonlinear Analysis: TMA*, *Numerical Functional Analysis and Optimization*, *Proceedings of the American Mathematical Society*, *Proceedings of the Royal Irish Academy*, and *ZAMM: Z. Angew. Math. Mech.* He is the founder and Editor-in-Chief of four journals from International Publications: *Advances in Nonlinear Variational Inequalities*, *Communications on Applied Nonlinear Analysis*, *Pan-American Mathematical Journal*, and *Transactions on Mathematical Programming and Applications*. He is also an Associate Editor of several international journals, including *Applied Mathematics and Computation*, *International Journal of Mathematics and Mathematical Sciences*, *Journal of Operators*, and *Journal of Computational Analysis and Applications*.