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Semi-Infinite Fractional Programming

Springer

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ISSN 2363-6149 ISSN 2363-6157 (electronic)
Infosys Science Foundation Series
ISSN 2364-4036 ISSN 2364-4044 (electronic)
Infosys Science Foundation Series in Mathematical Sciences
ISBN 978-981-10-6255-1 ISBN 978-981-10-6256-8 (eBook)
DOI 10.1007/978-981-10-6256-8

Library of Congress Control Number: 2017951997

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer Nature Singapore Pte Ltd.
The registered company address is: 152 Beach Road, #21-01/04 Gateway East, Singapore 189721, Singapore

Preface

This monograph is aimed at presenting a smooth and unified transition from the general fractional programming (or program) to the semi-infinite fractional programming (or program), especially in terms of framework for theoretical foundation and real-world applications, some new classes of generalized second-order invex functions and second-order univex functions, new sets of second-order necessary optimality conditions, numerous sets of second-order sufficient optimality conditions, and several sets of second-order duality models for establishing numerous duality theorems for a discrete minmax (or maxmin) semi-infinite fractional programming problem. Under the current interdisciplinary computer-oriented research environment, semi-infinite fractional programming is among the most rapidly expanding research areas in terms of its multifacet applications empowerment for real-world problems that can be handled by transforming them into semi-infinite fractional programming problems.

A mathematical fractional programming problem with a finite number of variables and infinitely many constraints is called a semi-infinite fractional programming problem in the literature. On the other hand, mathematical fractional programming problems with a finite number of constraints are referred to as generalized fractional programming problems, and they have been the focus of more research endeavors in terms of providing realistic models for some significant real-world problems, including the problems encountered in multiobjective programming, approximation theory, goal programming, facility location planning, and economics, and their mathematical tractability (they can be transformed into equivalent parametric nonlinear programming problems with nonfractional objective functions).

As a matter of fact, in mathematical optimization programs, a fractional programming (or program) is a generalization to linear fractional programming. These problems lay the theoretical foundation that will enable us to fully investigate the second-order optimality and duality aspects of our principal fractional programming problem as well as the semi-infinite counterpart, which is the main focus of this monograph to empower graduate students, faculty, and other research enthusiasts for more accelerated research advances with significant applications in the

interdisciplinary sense without borders. The main ingredients of this presentation are as follows:

Parametric Optimality
Parametric Duality
Parameter-free Optimality
Parameter-free Duality
Nonfractional Optimality
Nonfractional Duality

There are some quality books and surveys on generalized fractional programming problems, while it seems there are no such references exclusively in semi-infinite fractional programming other than some surveys in the literature since semi-infinite fractional programming is a new fast-developing research field transitioning from the generalized fractional programming. Furthermore, the methodology (the use of alternative concepts, partitioning schemes, and duality models) adopted in this book, as well as for the main results, will prove useful for other classes of nonlinear semi-infinite fractional programming and beyond. The generalized fractional programming problems have a wide range of real-world problems, which can be transformed into some sort of a generalized fractional programming problem.

Let us consider fractional programs that arise from management decision science. If we consider a system efficiency in an economical sense, it is equivalent to maximizing system efficiency leading to fractional programs with occurring objectives:

Maximizing productivity
Maximizing return on investment
Maximizing return/risk
Minimizing cost/time
Minimizing output/input

We envision that this monograph is a unique presentation of interdisciplinary research for the world scientific community (including graduate students, faculty, and general readers). Furthermore, some of the new concepts can be applied to duality theorems based on using a new class of multitime multiobjective variational problems as well.

I am extremely grateful to Prof. G.J. Zalmai, Northern Michigan University, for giving me tremendous opportunities for collaborative research, especially relating to semi-infinite fractional programming problems in a series of research publications, while most of the new concepts are already referred to as Zalmai-type sonvexities and Zalmai-type sounivexities in the literature. Furthermore, I express my deepest appreciation to Prof. R.N. Mohapatra, University of Central Florida, for his guiding star roles during my academic adventures and beyond.

San Marcos, USA Ram U. Verma
February 2017

Contents

About the Author

Ram U. Verma is President of International Publications, USA. Before joining Texas State University, he held several academic positions, ranging from Lecturer to Assistant Professor, Associate Professor, and Full Professor at the University of Cape Coast, the University of Tripoli, the University of Orient, the University of Puerto Rico, New York University (visiting faculty), the University of Central Florida, Mount Olive College, Duke University (visiting scholar), and the University of Toledo. His research interests encompass mathematical programming, fractional programming, semi-infinite fractional programming, multiobjective fractional programming, numerical analysis, generalized Newton's methods, new generation Newton-type methods, nonlinear functional analysis, applied analysis, evolution equations and semigroups, stochastic analysis, mathematics education, and determinant theory for singular integral equations.

He has published over 700 research articles in several international refereed journals including *Applied Mathematics and Computation, Applicable Analysis, Archivum Math, Communications in Nonlinear Science and Numerical Simulations, Czechoslovak Mathematical Journal, Electron, Journal of Differential Equations, Journal of Computational Analysis and Applications, Journal of Mathematical Analysis and Applications, Journal of Optimization Theory and Applications, Nonlinear Analysis: TMA, Numerical Functional Analysis and Optimization, Proceedings of the American Mathematical Society, Proceedings of the Royal Irish Academy,* and *ZAMM: Z. Angew. Math. Mech*. He is the founder and Editor-in-Chief of four journals from International Publications: *Advances in Nonlinear Variational Inequalities, Communications on Applied Nonlinear Analysis, Pan-American Mathematical Journal,* and *Transactions on Mathematical Programming and Applications*. He is also an Associate Editor of several international journals, including *Applied Mathematics and Computation, International Journal of Mathematics and Mathematical Sciences, Journal of Operators*, and *Journal of Computational Analysis and Applications*.