V-Invex Functions and Vector Optimization

Optimization and Its Applications

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Aims and Scope

Optimization has been expanding in all directions at an astonishing rate during the last few decades. New algorithmic and theoretical techniques have been developed, the diffusion into other disciplines has proceeded at a rapid pace, and our knowledge of all aspects of the field has grown even more profound. At the same time, one of the most striking trends in optimization is the constantly increasing emphasis on the interdisciplinary nature of the field. Optimization has been a basic tool in all areas of applied mathematics, engineering, medicine, economics and other sciences.

The series *Optimization and Its Applications* publishes undergraduate and graduate textbooks, monographs and state-of-the-art expository works that focus on algorithms for solving optimization problems and also study applications involving such problems. Some of the topics covered include nonlinear optimization (convex and nonconvex), network flow problems, stochastic optimization, optimal control, discrete optimization, multi-objective programming, description of software packages, approximation techniques and heuristic approaches.

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Preface

Generalizations of convex functions have previously been proposed by various authors, especially to establish the weakest conditions required for optimality results and duality theorems in nonlinear vector optimization. Indeed, these new classes of functions have been used in a variety of fields such as economics, management science, engineering, statistics and other applied sciences. In 1949 the Italian mathematician Bruno de Finetti introduced one of the fundamental generalized convex functions characterized by convex lower level sets, functions now known as "quasiconvex functions".

Since then other classes of generalized convex functions have been defined (not all useful at the same degree and with clear motivation) in accordance with the need of particular applications. In many cases such functions preserve some of the valuable properties of convex functions. One of the important generalization of convex functions is invex functions, a notion originally introduced for differentiable functions $f : C \to R$, C an open set of R^n, for which there exists some function $\eta : C \times C \to R^n$ such that $f(x) - f(y) \geq \eta(x, y)^T \nabla f(u),\ \forall x, u \in C$. Such functions have the property that all stationary points are global minimizers and, since their introduction in 1981, have shown to be useful in a variety of applications. However, the major difficulty in invex programming problems is that it requires the same kernel function for the objective and constraints. This requirement turns out to be a severe restriction in applications. In order to avoid this restriction, Jeyakumar and Mond (1992) introduced a new class of invex functions by relaxing the definition invexity which preserves the sufficiency and duality results in the scalar case and avoids the major difficulty of verifying that the inequality holds for the same kernel function. Further, this relaxation allows one to treat certain nonlinear multiobjective fractional programming problems and some other classes of nonlinear (composite) problems. According to Jeyakumar and Mond (1992) A vector function $f : X \to R^p$ is said to be *V*-invex if there exist functions $\eta : X \times X \to R^n$ and $\alpha_i : X \times X \to R^+ - \{0\}$ such that for each

$x, \bar{x} \in X$ and for $i = 1,2,\ldots, p$, $f_i(x) - f_i(\bar{x}) \geqq \alpha_i(x,\bar{x}) \nabla f_i(\bar{x}) \eta(x,\bar{x})$. For $p = 1$ and $\bar{\eta}(x,\bar{x}) = \alpha_i(x,\bar{x}) \eta(x,\bar{x})$ the above definition reduces to the usual definition of invexity given by Hanson (1981).

This book is concerned about the V-invex functions and its applications in nonlinear vector optimization problems. As we know that a great deal of optimization theory is concerned with problems involving infinite dimensional normed spaces. Two types of problems fit into this scheme are Variational and Control problems. As far as the authors are concerned this is the first book entirely concerned with V-invex functions and their applications.

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